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Quantitative Measures for Cartogram Generation Techniques

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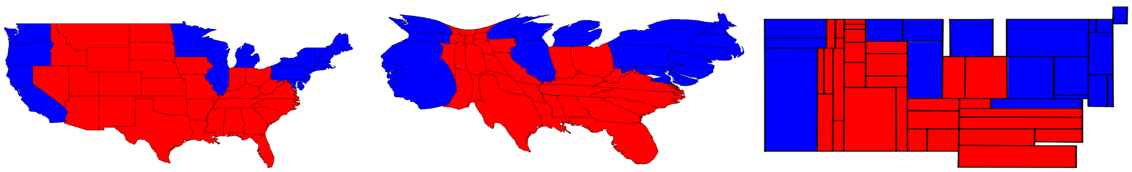

(a) (b) (c)

Figure 1: *2004 US Presidential elections: (a) geographically accurate map, (b) diffusion cartogram, (c) rectangular cartogram.*

Abstract

Cartograms are used to visualize geographically distributed data by scaling the regions of a map (e.g., US states) such that their areas are proportional to some data associated with them (e.g., population). Thus the cartogram computation problem can be considered as a map deformation problem where the input is a planar polygonal map M and an assignment of some positive weight for each region. The goal is to create a deformed map M', where the area of each region realizes the weight assigned to it (no cartographic error) while the overall map remains readable and recognizable (e.g., the topology, relative positions and shapes of the regions remain as close to those before the deformation as possible). Although several such measures of cartogram quality are well-known, different cartogram generation methods optimize different features and there is no standard set of quantitative metrics. In this paper we define such a set of seven quantitative measures, designed to evaluate how faithfully a cartogram represents the desired weights and to estimate the readability of the final representation. We then study several cartogram-generation algorithms and compare them in terms of these quantitative measures.

1. Introduction

A *cartogram*, or value-by-area diagram, is a thematic visualization of a planar map, where geographic regions such as countries or provinces are modified in order to realize a given set of values by their areas. This kind of visualization has been used for many years to represent census data such as, population or gross-domestic-product, and to visualize election returns, disease incidence and other geo-referenced statistical data. Red-and-blue population cartograms of the United States are often used to illustrate the results in presidential elections starting in the year 2000. For example, in the 2004 elections, geographically accurate maps seemed to show an overwhelming victory for George W. Bush, while the population cartograms effectively communicate the near even split, by deflating the rural and suburban central states; see Fig. 1. Cartograms are also widely used in newspapers and social media [Gua12, LAT12, NYT08], textbooks [Car14, Pel], and TED talks [Mil08, Ros06] to show geographic, political and socio-economic data; see [NK15] for more details.

Incorporating vastly different scaling factors for different states could force significant topological or geometrical distortions in the input map, resulting in poor readability and recognizability for the map. This is undesirable for effective visualization of the given data, since the cartogram should enable the viewer to quickly relate the displayed data to the original map. This recognizability depends on preserving basic properties such as shapes and relative positions or orientations for the regions, as well as the basic topology of the map. All of these goals are difficult to achieve simultaneously, and in general, it is impossible to retain the original map's topology and shapes of the regions, while realizing the geo-referenced data perfectly; Fig. 2 shows an artificial example; also see [dBMS10, KNP04]. In the less artificial example of the rectilinear cartogram in Fig. 1(c), the correct distribution of red and blue areas are shown, but several characteristic shapes and adjacencies are compromised: Oklahoma and New Mexico no longer share borders; the mirror-image shapes of New Hampshire and Vermont are lost.

None of the existing algorithms and techniques to gener-

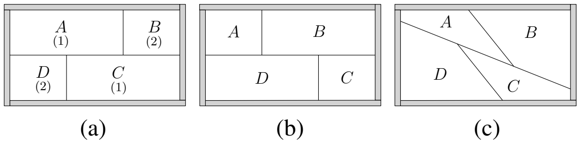

Figure 2: *Three cartograms for a map with an area assignment for four states (A, B, C, D with desired areas* 1, 2, 1, 2, *respectively), containing (a) cartographic error, (b) topology error and (c) shape error. There is no cartogram with no cartographic error, topology error and shape error.*

ate cartograms for a map is "perfect"; each of them produces a "good" cartogram with respect to some criteria, but which might be "bad" with respect to others. Although several such measures of cartogram quality are well-known, different cartogram generation methods optimize different features and there is no standard set of quantitative metrics. In this paper we propose a set of quantitative measures that can be used to estimate the accuracy of a cartogram in realizing the geo-referenced weights, in maintaining the original map properties in the final representation, and well as the efficiency of cartograms in terms of time complexity and the polygonal complexity of the represented regions.

1.1. Related Work

Showing statistical information on top of a geographic map is a common goal in geo-visualization and there are two different approaches. *Augmented map visualizations* have multiple correlated views, showing the geographic map, and data plots (such as histograms, pie-charts and scatter-plots) are used to show some geospatial data [FS04, MBHP98]. One advantage of this approach is that both univariate and multivariate data can be visualized. One disadvantage is the weak connection between the data and the geospatial locations, due to relatively weak association between the plots and the geographic map. In an interesting variant, the augmentation projects the regions of the map into a different geometric structure. Recent examples of such maps include *grid map layouts* [EvKSS13], where the regions of the maps are projected onto a square grid by means of point-set matching, *table cartograms* [EFK*13], where the grid cell areas match pre-determined areas, and *necklace maps* [SV10], where the regions of a map are projected onto intervals on a one-dimensional curve that surrounds the map regions.

Unlike in the augmented maps visualizations, in *cartograms* the geospatial data and the map topology are shown by a single visualization. There are two major types of cartograms. In the first type, *deformation cartograms*, the input map itself is modified by appropriately pulling and pushing boundaries to change the areas of the regions on the map. In the second type, *topological cartograms*, the topology of the map is extracted in the form of the dual graph, and the dual graph is used to obtain a schematized layout for the map. Here the map regions are highly schematized and have very small (often constant) polygonal complexity. Both types have their advantages and disadvantages. Since deformation cartograms are formed by a continuous deformation of the map, the map often remains recognizable; while for topological cartograms recognizability is difficult to achieve. On the other hand, since topological cartograms have small polygonal complexity, it is easier to compare and contrast the areas of different regions.

Among deformation cartograms, the most popular method is the diffusion-based algorithm of Gastner and Newman [GN04], where the original input map is projected onto a distorted grid, computed in such a way that the areas of the countries match the pre-defined values. Dougenik *et al.* introduce a method based on force fields where the map is divided into cells and every cell has a force related to its data value which affects the other cells [DCN85]. Dorling uses a cellular automaton approach, where regions exchange cells until an equilibrium has been achieved, i.e., each region has attained the desired number of cells [Dor96]. It is worth-mentioning that this technique can result in significant distortions, thereby reducing readability and recognizability, which is usually one of the main advantages of this type of cartogram. Welzl *et al.* generate cartograms using a sequence of homeomorphic deformations and measure the quality with local distance distortion metrics [WEW97]. Kocmoud and House [HK98] describe a technique that combines the cell-based approach of Dorling [Dor96] with the homeomorphic deformations of Welzl *et al.* [WEW97]. Keim *et al.* describe deformation algorithms CartoDraw [KNP04] based on incremental repositioning of the vertices of the map's polygons by means of local changes of horizontal and vertical scan lines, and VisualPoints [KPN05] based on quadtree partitions of the plane. In a variant of Cartodraw, the medial axes for the polygonal regions are used as the scanlines [KPN05]. Although also deformation-based, the method by Kämper *et al.* [KKN13] is different in that it uses only circular arcs. Here the straight-line segments of the map are replaced by circular arcs so that the countries with less area in the original map than required inflate (and become cloud-shaped), while those with more area than required deflate (and become snowflake shaped). Thus in such a cartogram, it is easy to determine whether a country has grown or shrunk, just by its overall shape.

Topological cartograms date back to the 19th century and the highly schematized rectangular cartograms of Raisz [Rai34], where each country is represented by an axis-aligned rectangle. Several more recent methods for computing rectangular cartogram have been proposed [KS07, HKPS04, BSV12]. The main advantage of such rectangular representations is that it is usually easy to compare the areas for the regions in the map, unless the rectangles have poor aspect ratio. One disadvantage is that with rectangular cartograms, it is not always possible to maintain the topology of the map, i.e., not all given pairwise adjacencies between countries can be maintained. Thus in all the exist-

Criteria	Measure & Notation		Definition
Statistical Distortion	**Average Cartographic Error**	ε	$\frac{1}{\lvert V\rvert}\sum_{v\in V}\frac{\lvert o(v)-w(v)\rvert}{max\{o(v),w(v)\}}$
	Maximum Cartographic Error	ξ	$\max_{v\in V}\frac{\lvert o(v)-w(v)\rvert}{max\{o(v),w(v)\}}$
Topology Distortion	**Adjacency Error**	τ	$1-\frac{\lvert E_c\cap E_m\rvert}{\lvert E_c\cup E_m\rvert}$
Orientation and Shape Distortion	**Angular Orientation Error**	θ	[HKPS04]
	Hamming distance	δ	[Ski97]
	Average Aspect Ratio	α	[KS07]
Complexity	**Polygonal Complexity**	η	Maximum Number of Corners per Region

Table 1: *Definition of Performance Measures.*

ing methods for rectangular cartogram, some adjacencies are compromised, or a few countries are allowed to have non-rectangular shapes, or some errors in the representation of weights by area are allowed. Relaxing the requirement of rectangular shape, some algorithms use more general rectilinear shapes (still of constant polygonal complexity) to produce error-free cartograms. The upper bound on the complexity of the polygonal shapes used in these cartograms has been reduced from the initial 40 [dBMS09] to 34 [KN07], 12 [BV11], 10 [ABF*13a] and finally to 8 [ABF*13b], which also matches the lower bound [YS93]. There is also an algorithm that modifies a rectangular cartogram into a rectilinear one with a guarantee of no error and in practice most regions continue to have very small complexity [dBMS10]. There are many more algorithms to generate cartograms; see a great survey by Tobler for more information [Tob04].

Cartograms can be evaluated in terms of the error in realizing the desired value by area, and in terms of the preservation of the recognizability of the input map. There are several algorithms for which one or two desirable cartogram features have been evaluated quantitatively [BSV12, dBMS10,KS07,KNPS03]. The cognitive perception of static cartograms has been considered by Dent [Den75] and Griffin [Gri83] and interactive cartograms were evaluated by Ware [War98].

1.2. Our Contributions

In this paper we study various quantitative measures that have been used in the literature for the analysis of cartogram algorithms. We compare how well these measures capture different properties of cartograms. Based on this analysis, we propose a set of quantitative measures (see Table 1) that we use to compare several cartogram algorithms, and which might be useful in future quantitative evaluation of cartograms.

2. Performance Measures for a Cartogram

The challenge in creating a good cartogram is to shrink and grow the regions in a map so that they reflect the set of pre-specified area values (faithful realization of the weight), while still retaining their characteristic shapes, relative positions, and adjacencies (faithful representation of the map). As we have already seen in Fig. 2, there are trade-offs between these two goals. For efficient rendering of cartograms, polygonal complexity and running time are also sometimes important considerations. Here we consider several quantitative measures that capture different properties of the cartograms, some already measured in various earlier work, while others not yet quantified. We analyze new and existing measures to compare how faithfully they represent the intuitive notions for the criteria they measure. Tobler [Tob04] suggested that cartogram algorithms should be evaluated for (i) correct realization of weights, (ii) preservation of region shapes to the extent possible, and (iii) efficient running time, in this order. We analyze the quantitative measures exactly in these three categories: (i) parameters that measure *statistical distortion*, i.e., the degree of inaccuracy in the value-by-area realization of the statistical data, (ii) parameters that capture *geographic distortion*, i.e., how deformed is the cartogram compared with the original map, and (iii) an estimation of the *complexity* of cartogram in terms of the running time and the polygonal complexity of the regions in the representation. Based on this analysis we select a set of seven standard measures that we use in this paper, and that will hopefully be used in future evaluations; see Table 1.

We assume an input map M is partitioned into n countries with polygonal boundaries. For each country v, $a(v)$ denotes the area of v in M and the weight $w(v)$ is the desired area for the country. Both a and w are normalized to the same total area, i.e., $\sum_{v\in V} a(v) = \sum_{v\in V} w(v)$. An algorithm then attempts to construct a cartogram M', that is a deformation of M, where each country v obtain $o(v)$ area. We now define some quantitative measures.

2.1. Parameters for Statistical Distortion

The most common measure for distortion in the value-by-area realization is the *cartographic error*. Even though intuitively clear, this has been defined in different ways, which often make sense for the specific algorithm under consideration, but not necessarily for all cartogram algorithms.

Cartographic Error: Given a cartogram, the *individual cartographic error* for each country v is defined as the value of $|o(v)-w(v)|$ where $o(v)$ and $w(v)$ are the obtained and required areas for the country [KNPS03]. This value is generally *normalized* and the *overall cartographic error* for the cartogram is obtained by combining these individual normalized errors for all countries. We consider three different normalization factors: (i) the required area $w(v)$, as in [KS07, BSV12], (ii) the summation $o(v)+w(v)$, as in [KNP04], and (iii) the maximum of $o(v)$ and $w(v)$. The

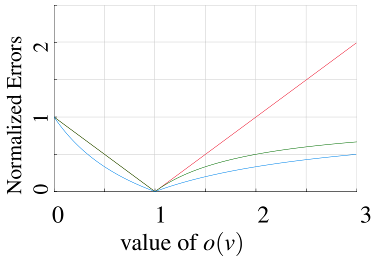

Figure 3: *Normalized cartographic errors with different obtained areas $o(v)$ for a country v with desired area $w(v) = 1$ unit. Red, blue and green curves give the distributions when $w(v)$, $o(v)+w(v)$, and $\max\{o(v), w(v)\}$ are the normalization factors.*

first one is asymmetric with respect to $o(v)$ and $w(v)$; while the other two are symmetric. It is preferable to normalize the error by some symmetric function of $a(v)$ and $w(v)$; otherwise, if we use $w(v)$ as the normalization factor, then the normalized error penalizes a country that needs to grow and a country that needs to shrink in asymmetrically; see Fig. 3.

In particular, for a country v for which $w(v) \geq o(v)$ the error lies in the range $[0,1]$, while in case $w(v) \leq o(v)$, the error lies in the range $[0,\infty)$. Thus when we combine these individual normalized errors, the result depends on the number of countries that need to grow or shrink, rather than the amount by which they need to grow or shrink. This may lead to undesirable and counter-intuitive behavior; see Fig. 4.

From the two symmetric normalization functions the maximum of $o(v)$ and $w(v)$ seems better as it gives a more uniform distribution of cartographic error (see Fig. 3, specifically, for $o(v) = w(v)/2$, i.e, when some country has half the desired area; the error should intuitively be 0.5, but using $o(v) + w(v)$ as the normalization factor, leads to an error of 0.3). Finally, there are two standard ways in the literature to combine individual errors in order to compute the overall cartographic error: (i) by taking the average (or equivalently summation) for the individual errors (e.g., [KNPS03]), or (ii) by taking the maximum for the individual error (e.g., [KS07]). The maximum cartographic error is much more sensitive to "outliers" than the average. For example, consider a country with 100 states, where the desired area for each is 1 unit and each achieves the correct area in a cartogram except one which has area 101 units. Then after normalization, the maximum and the average cartographic errors are 0.98 and 0.55, respectively.

We therefore use both the maximum and the average as they capture different aspects of statistic distortion. The average normalized cartographic error is given by $\varepsilon = \frac{1}{|V|} \sum_{v \in V} (|o(v) - w(v)|)/(max\{o(v), w(v)\})$. The maximum normalized cartographic error is given by $\xi = \max_{v \in V}(|o(v) - w(v)|)/(max\{o(v), w(v)\})$; see Table 1.

Kämper *et al.* [KKN13] use *success rate* to evaluate the realization of weights. In addition to the obtained and required areas, this measurement uses the original area of the

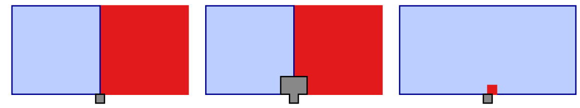

Figure 4: *Cartograms with no cartographic error realizing areas (100, 100 and 1 units) (left), with symmetric errors (97,97,7 units) (center), and with asymmetric errors (199,1,1 units) (right). Intuitively, the center map is more accurate than the right map as it still has one small and two large regions. This intuition is supported by symmetric normalization (0.3 for center, 0.495 for right), while the asymmetric one (2.02 for center, 0.66 for right) reverses the order.*

countries in the given geographic map. The goal is to evaluate the achieved area change, relative to the required area change. Thus the success rate for a country v is $|\frac{o(v)-a(v)}{w(v)-a(v)}|$, where $a(v)$ is the actual area of v in the input map. This measure is natural for cartograms that are generated by gradually deforming the input map to realize the weights. However, for cartograms generated using the topology or the dual graph, the actual area of a country in the original map does not play any role (as only the topology matters).

2.2. Parameters for Geographic Distortion

In order for a cartogram to effectively visualize some given data, such as population or GDP, it is important that the cartogram is *readable*, in that one can find and identify every country, and *recognizable*, in that one can see the same structure and topology as in the input map. Thus the measures for geographic distortion can be subcategorized in two classes: (i) those that compute how much the topology (region-adjacencies) changed, (ii) those that estimate distortions in the shapes and relative positions of the shapes.

Topology/Adjacency Distortion: The adjacency error τ is an estimation of how the adjacency relationships between pairs of neighboring countries have been affected in the cartogram, compared to the original map. Similar to [HKPS04] we measure this by the fraction of the adjacencies that the cartogram fails to preserve; i.e., $\tau = 1 - \frac{|E_c \cap E_m|}{|E_c \cup E_m|}$, where E_c and E_m are respectively the adjacencies between countries in the cartogram and the map.

Some cartogram algorithms suffer from an even more serious disadvantage. Sometimes to make the input map suitable for the algorithm, some regions on the map is deleted or merged with other neighboring regions. In the context of cartograms, this is unacceptable and should be avoided.

Relative Position/Orientation: Preserving the relative positions of different countries in the map is important for the recognizability of a cartogram. To quantify this notion, Buchin *et al.* [BSV12] introduced the *bounding box separation metric*, which measures the average distance by which the bounding boxes for pairs of countries move from the original map to the cartogram. This measure is well-defined for rectangular layout but cannot be easily extended to general cartograms. In particular, the bounding boxes are always

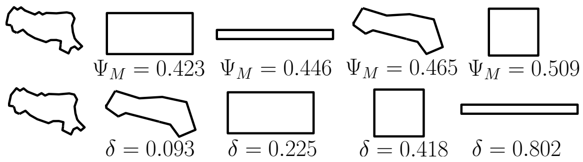

Figure 5: *A polygonal region of a map and four of its schematizations sorted in decreasing order by* turning angle distortion Ψ_M *(top) and by* Hamming distance δ *(bottom). Intuitively, the bottom order is more accurate as it seems to capture the notion of the original shape better than the top one. Note that* Ψ *and the modified variant* Ψ_M *gives the same ranking (top).*

disjoint in rectangular cartogram, while this is not the case for general cartograms. This takes into account only the relative position of adjacent countries, while adjacencies only partially reflect the notion of relative positions [HKPS04].

To estimate the relative positions distortion, we use *angular orientation error*, θ, defined by Heilmann *et al.* [HKPS04] and obtained by computing the average change in the slope of the line between the centroids of pairs of countries. For applications where only the orthogonal relative position (north-south, east-west) is important, this measure θ can be approximated by calculating the fraction of pairs of countries for which the relative north-south and east-west orientations have changed. Note that this measure is also a generalization of the binary variant for the bounding box separation metric defined by Buchin *et al.* [BSV12].

Shape Distortion: We need to measure how the shape of a country in the generated cartogram compares with its original shape in the input map. Arkin *et al.* [ACH*90] compute the deviation between two polygons by normalizing them by perimeter and then measuring a turning function, which captures turning angle and edge length. This measure is translation-invariant, scale-invariant, and rotation-invariant. Thus two rectangles which are the same, up to rotation, are considered identical with this measure. Keim *et al.* consider a similar measure (also translation-invariant, scaling-invariant, and rotation-invariant) for shape comparison between countries [KNPS03] based on a Fourier transformation of the turning angle functions. At the other end of the spectrum of shape distortion measures, Heilmann *et al.* [HKPS04] uses only the aspect ratios of the axis-aligned bounding boxes when comparing the shapes of polygons.

None of these functions captures faithfully the similarity between the shapes of countries on a map. Furthermore Arkin *et al.* [ACH*90] pointed out that the turning function is sensitive to non-uniform noise, which makes it undesirable to compare geographic shapes. Intuitively, a measure that is translation-invariant, scale-invariant, but not rotation-invariant would be better, since the orientation is an important feature for distinguishing between countries in a map. With this in mind we consider three shape distortion measures: (1) *turning-angle distortion*, Ψ by Arkin *et al.* [ACH*90], (2) *modified turning-angle distortion*, Ψ_M, where we have removed its rotation-invariance, and (3) *Hamming distance*, δ. The last one is based on the idea of Hamming distance [Ski97] or symmetric difference [MvRS10] between two polygons. Two polygons are superimposed on top of each other and the fraction of the area that is in exactly one of the polygons is measured. In order to make the comparison scale-invariant, we normalize the area of polygons to unit area. To make it translation-invariant we consider all possible values of translation up to a small discretization and use the one that gives the smallest error. We compared the three measures using several real-world and synthetic examples. Our results indicate that the *Hamming distance*, δ best captures the notion of similarity between the shapes of countries on a map; see Fig. 5.

Aspect Ratio: Especially in rectangular cartograms, the *aspect ratio*, is an important factor for readability and poor aspect ratios make it difficult to show labels [KS07]. We measure the aspect ratio, α, by computing the ratio between the larger and smaller sides of the bounding box for each region in the map, and taking the average.

2.3. Complexity Measures

These parameters are related to the efficiency of rendering cartograms and the visual complexity of the regions.

Polygonal Complexity: There are practical and cognitive reasons to limit the polygonal complexity of regions in cartograms. For some cartograms, such as rectangular [BSV12] and some rectilinear cartograms (e.g., [ABF*13b]), each polygon has a constant complexity. For some others, such as diffusion-based cartograms [GN04] and circular-arc cartograms [KKN13], the polygonal complexity of the regions in a cartogram depends on the input map. In addition to giving more visually complex shapes, high polygonal complexity also results in significant increase in size (e.g., for maps of the USA, 400KB for diffusion-based cartogram [GN04] vs 8KB for rectangular cartogram [BSV12]). As in [dBMS10], we measure this in terms of maximum and average polygonal complexity of the map regions.

Running Time: For cartogram systems, in particular for interactive cartogram software, fast computation is essential. The running time is thus an important factor for evaluating cartogram algorithms.

3. Cartogram Algorithms

Here we review several cartogram generation methods (diffusion-based, circular-arc, rectangular, rectilinear). We implemented all of them and used many different input maps and statistics to generate different cartograms. We will illustrate how our proposed measures can be used to compare these very different methods.

3.1. Deformation Cartogram Algorithms

Diffusion Method (*DIFF*): Gastner and Newman use a diffusion method for creating cartograms [GN04], where the original input map is projected onto a distorted grid, computed in such a way that the areas of the countries match exactly the pre-defined values. This method uses a physical model in which the desired areas are achieved via an iterative diffusion process, where flows move from one country to another until a balanced distribution is reached. After each iteration the new coordinates for points on the map are computed by interpolation of the distorted grid points. The cartographic error, shape distortion and running time for this method depends on the size of the chosen grid. We have used a 1024×1024 grid that generally is sufficient for the geographic maps under consideration.

Circular-Arc Cartogram Algorithm (*CIRC*): The method of Kämper *et al.* [KKN13] deforms a given geographic map, using circular arcs instead of straight-line segments along the border between two regions, in order to grow and shrink the regions on the map. First a flow network is computed from the dual graph of the map, where bidirectional edges are created between pairs of adjacent countries. The flow on such edge (a,b) represents the area that the polygon for a transfers to the polygon for b. The capacity of this edge is assigned as the maximum "safe" area that can be transferred from polygon for a to polygon for b, without creating any crossing or overlapping polygons. Each country that needs to shrink (grow) is connected to a source (sink) node and the capacity on these edges corresponds to how much these countries need to grow or shrink. The output cartogram is the one that maximizes the flow in the network.

We make two modifications in the algorithm by Kämper *et al.* [KKN13] to improve its performance. First, the original algorithm did not allow the area of the "sea" to change. As a result a country cannot grow into the sea unless there is some other country that can compensate for it by shrinking near the sea.We overcome this by augmenting the flow network: in addition to the edges of the original flow network, we add high-capacity small-weight edges from the source to the sea and from the sea to the sink to allow the sea area (equivalently, total land area) to be changed. Second, we note that if the boundary of a country only contains line-segments of small length, then it is impossible to achieve significant change in the area by replacing the straight-line segments with circular arcs. We therefore carefully remove intermediate degree-2 points on the boundary of a region, thus allowing significant change in the area, while still preserving the overall shape of the region. We accomplish this with a modified version of the poly-line simplification algorithm of Douglas and Peucker [DP73]. We use this strategy iteratively, where at each iteration we select a border between pair of countries that gives the maximum reduction in cartographic error and simplify the border between them by removing half of the degree-2 points.

3.2. Topological Cartogram Algorithms

Rectangular Cartogram Algorithms (*RECT*): Not all planar maps can be drawn so that all the countries are rectangles. But if we tolerate some topological errors, it is possible to compute a rectangular cartogram. Such cartograms were studied by Van Kreveld and Speckmann [KS07]. They used three different heuristics for computing rectangular cartograms. We implement their "segment-moving heuristic" to generate cartograms. This heuristic gives two different methods: in one the adjacencies might be disturbed to realize the weights perfectly (*area-preserving*); in the other all the adjacencies are maintained but the cartograms would contain some cartographic errors (*topology-preserving*). While the area-preserving variant converges fast to a cartogram with zero cartographic error in the simple segment-moving heuristic, the topology-preserving cartograms have significant cartographic error. Hence for the later variant, we use the improved algorithm by Buchin *et al.* [BSV12].

The evolution algorithm in [BSV12] generates the "fittest" rectangular cartogram for a planar map. At each step the algorithm takes a number of different rectangular layouts for the map and keeps only those for which the cartogram computed using the approach in [SKF06] gives the least error or the best "score" for a given scoring function. Then a number of new rectangular layouts are generated by combining the "fittest" old ones. Since this algorithm produces topological cartograms which are always inferior to deformation cartograms in terms of recognizability and shape preservation we use a scoring function that optimizes the cartographic error, which is the strongest feature of these cartograms. We also only consider the topology-preserving variant since in the area-preserving variant, the segment-moving heuristic gives zero error. We call these two variants *RECT-A* (area-preserving segment-moving heuristic) and *RECT-E* (topology-preserving evolution algorithm).

Note that in both rectangular cartograms, the shape of the outside boundary is manually determined by placing a set of rectangles for the "sea" regions. Careful placement of the sea regions can lead to better performance [BSV12, KS07]. To allow for the comparison between different algorithms we use a standardized sea-procedure. We always place exactly four sea regions in the left, top, right and bottom borders of the map. In order to make a map realizable with rectangular cartogram, sometimes these methods merge two countries into one or split some country into two parts, which may be undesirable. For example, in a cartogram of Europe, the region for Luxembourg either gets merged with one of its neighboring countries or one of the neighbors of Luxembourg gets split into two parts [KS07]. In practice, this results in regions that are no longer rectangular, but still have low polygonal complexity.

T-Shape Cartogram Algorithm (*COMBT*): Using a Schnyder realizer [Sch90] and the area-universality of one-sided rectangular duals [EMSV12], one can compute rectilinear cartograms with optimal complexity [ABF*13b]. Each country in the resulting cartogram is drawn by a T-shape with at most 8 corners per polygon and the desired

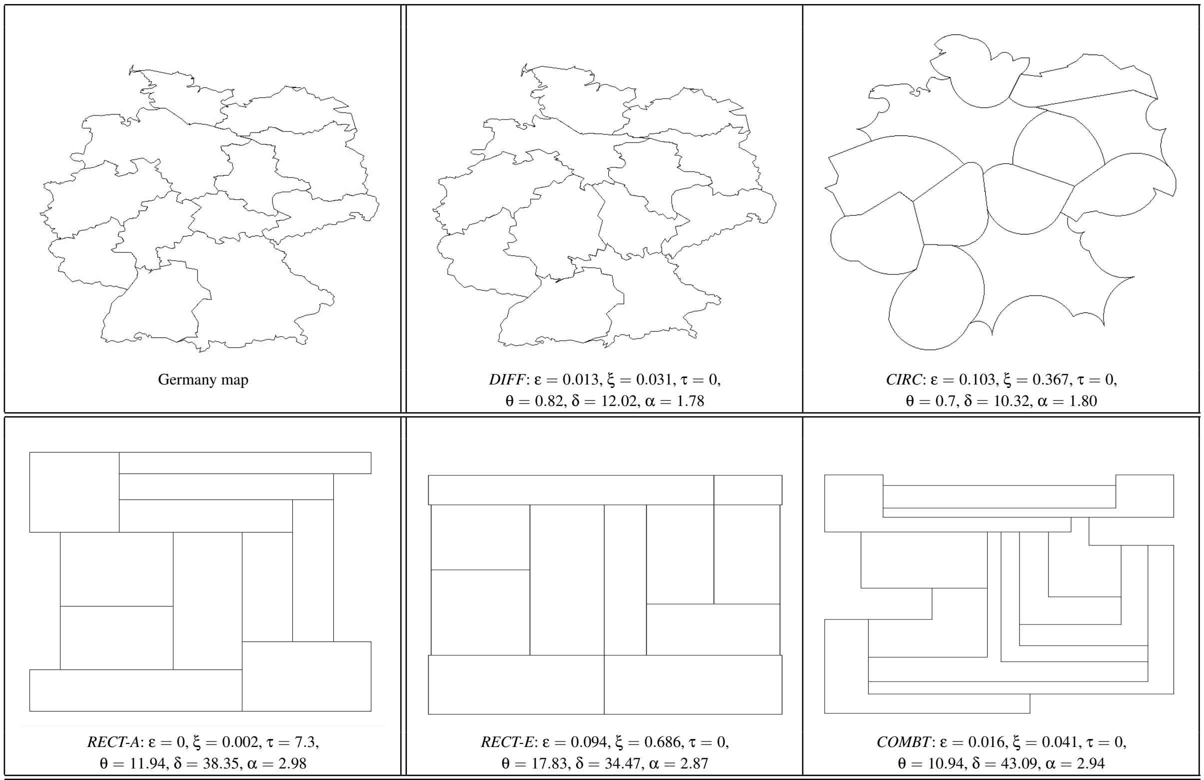

Figure 6: *A map of Germany and GDP cartograms by different algorithms, along with the values for average and maximum normalized cartographic error (ε and ξ), topology distortion τ, angular orientation error θ, Hamming distance δ and average aspect ratio α.*

areas are obtained via an iterative process that mimics the natural phenomenon of air-pressure. This method of producing cartograms guarantees convergence to an error-free cartogram and converges quickly in practice.

Country	Number of States	Number of Dual Edges	Average Polygon Complexity
USA	46	117	52.4
Germany	12	28	66.3
Italy	15	30	32

Table 2: *Properties of the input maps.*

Country	Data	Maximum Value	Minimum Value	Average Value
USA	GDP	1936400	26400	284846.7
	Population	38041430	576412	5926901.7
Germany	GDP	37509	21404	27822.9
	Population	17837000	1639000	6232500
Italy	GDP	321627	6067	82289.53
	Population	9642000	320000	3377196.6

Table 3: *Properties of the input data.*

4. Evaluation using Proposed Metrics

Here we show how to use the defined standard measures for statistical distortion, geographic distortion, and complexity in order to analyze the performance of the different cartogram algorithms. We begin with a description of the maps and data that we used for our experiments.

4.1. Datasets and Experimental Settings

We use maps of the USA, Italy and Germany to compare all the cartogram algorithms. For each map we use GDP and population data for 2010. All the maps and datasets are available at `http://cartogram.cs.arizona.edu/data.html`. The properties of these maps and the data are shown in Tables 2 and 3. After each algorithm is run on each map and dataset, we compute the value of the performance measures ε, ξ, τ, θ, δ and α. We also record the running time and polygonal complexity (either constant or equal to the input for all the algorithms considered). All experiments were performed on on an Intel Core i5 1.8GHz machine with 8 GB RAM. GDP cartograms of Germany for the five algorithms are shown in Figure 6.

4.2. Performance Analysis

We analyze the performance of the five cartogram algorithms with the three datasets and two statistics, using our proposed metrics.

Deformation Algorithms vs Topological Algorithms: Recall that all the algorithms under consideration fall into

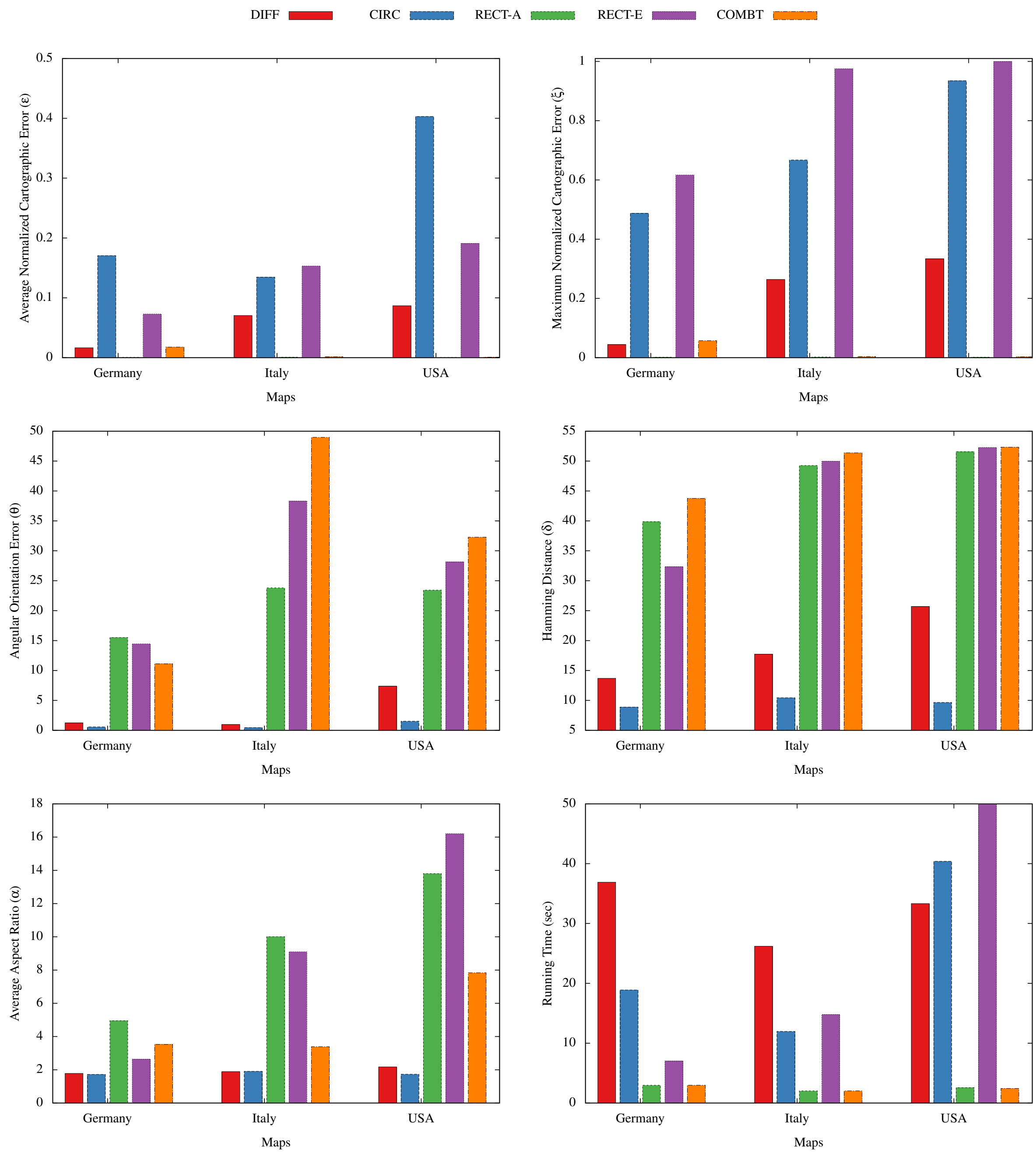

Figure 7: *Average normalized cartographic error* ε*, maximum normalized cartographic error* ξ*, angular orientation error* θ*, Hamming distance* δ*, average aspect ratio* α *and running time for all algorithms (the iterative algorithm RECT-E was stopped after 50 seconds).*

two types: deformation algorithms (*DIFF* and *CIRC*), and topological algorithms (different variants of *RECT* and *COMBT*). The deformation algorithms modify the input map by either moving vertices or by deforming edges, while the topological algorithms use the map topology (i.e., the planar dual) and construct cartograms exploiting different combinatorial properties. Since deformation algorithms work on the map itself rather than the dual, they produce cartograms with better readability and recognizability than topological algorithms; which is also evident from the value of the measures (θ, δ, α) related to map recognizability; see Fig. 7.

However, topological algorithms have the advantage of constant polygonal complexity, while for deformation algorithms the polygonal complexity is the same as the input map. Furthermore some of the topological algorithms (*RECT-A*, *COMBT*) also guarantee zero cartographic error for any map and input data; see Fig 7.

Diffusion Method vs Circular-Arc Cartograms: Among the deformation algorithms, the circular-arc cartograms perform slightly better than the diffusion method in terms of the three readability measures (θ, δ, α); see Fig. 7. This suggests that circular-arc cartograms preserve the state-shapes and their relative positions better. On the other hand the diffusion method generates cartograms with lower cartographic error. While the errors for circular-arc cartograms are comparable with (slightly worse than) *RECT-E*, they are much worse than *COMBT* and *RECT-A*.

Comparison Among Topological Algorithms: Among the topological algorithms, the *COMBT* algorithm and the *RECT-A* algorithm achieve almost zero cartographic error (they eventually converge to cartograms with zero cartographic error), but the *RECT-A* algorithm significantly distorts topology (missing 3-7 pairwise adjacencies in the resulting cartograms). All *RECT* variants require that one or more regions be deleted or merged with neighboring countries, in order to make the map suitable for rectangular representation. In particular, these three algorithms delete one state from Italy, two states from Germany and four states from the USA to make the graph 4-connected, which guarantees the existence of such a rectangular drawing. This can be avoided by allowing some regions to have more complex shapes than rectangles (e.g., rectilinear). *RECT-E* generates rectangular cartograms with much higher cartographic error; see Fig. 7(right), but unlike *RECT-A* it preserves adjacencies between regions.

Time-Analysis: Most of the algorithms generate cartograms fairly quickly but not all are suitable for real-time cartogram generation; see Fig. 7. The *COMBT* and the *RECT* algorithms guarantee convergence to zero cartographic error eventually; running them for 1–3 seconds suffices to achieve negligible error. The running time for both the iterative algorithms (*CIRC*, *RECT-E*) depends on the number of iterations. We run them for 20–30 iterations, while the average time per iteration for these algorithms is about 2–5 seconds on USA map and about 1 second for the Germany and Italy maps. The running time for *DIFF* is about 25–50 seconds.

Overall Comparison: It is clear that there is no single cartogram algorithm that satisfies all the desirable criteria. Some can guarantee very low *statistical distortion*. Others can provide very low *geographic distortion*. The algorithms also differ in complexity (both in terms of running time complexity and polygonal complexity of the regions). Thus which cartogram algorithm should be used in practice depends on the application requirements and on the viewers' preferences. For example if the only criteria for a cartogram is zero cartographic error, then *RECT-A* seems very good. If in addition, topology is to be preserved, *COMBT* might be a preferred choice. Further, if readability of the map is very important, then one might choose *DIFF* or *CIRC*, while if reasonable cartographic error and readability but low polygonal complexity is needed, *RECT-E* may be suitable.

5. Conclusion, Limitations and Future Work

We propose a set of quantitative measures as a standard to evaluate cartogram generation algorithms in terms of correct representation of statistical data, faithful realization and readability of underlying map, and complexity of the cartogram and the cartogram generation method. To the best of our knowledge this is the first attempt to standardize these measures, although the necessity of such measures has been expressed before [Tob04]. We also compare five different cartogram algorithms, using these measures. However, the purpose of our experiment is only to show how the quantitative measures can be used to evaluate cartogram algorithms. The results obtained in our experiments are only applicable to the relatively small dataset (3 countries, 2 statistics per country) and it was not our intention to conduct a full-scale comparison between these algorithms. A full-scale experiment with more countries, more statistics, and with more cartogram algorithms is left for future work. Further, in order to validate the proposed quantitative measures for cartogram algorithms, it is necessary to perform qualitative analysis (user-study) and verify that the proposed measures indeed capture the desired features.

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